EXTERNALITIES AS A NECESSARY CONDITION FOR CYCLICAL SOCIAL PREFERENCES

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I. INTRODUCTION

Ever since Arrow put forward his General Impossibility Theorem, it has been well-known that no consistent Social Welfare Function can exist in a non-oligarchic society if certain reasonable conditions are fulfilled [Arrow, 1963]. It has been shown, moreover, by Sen [1970] that a dilemma exists for "Minimal Liberalism" in the sense that it leads to a contradiction with Pareto optimality. These results demonstrate that several inconsistencies and dilemmas are connected with different decentralized organizational forms of society.

Another part of the literature is concerned with market failures. Here it has been shown that a market organization of the economy can, even under a regime of perfect competition, lead to non-Pareto optimal outcomes, if externalities are present [Mishan, 1971]. Again it appears obvious that there exists some relationship between this result and the type of organization of society. But this time not only the kind of organization, but also the presence of externalities is necessary for the result. One wonders, therefore, whether there might exist a relationship between the results of the two fields of economic literature. To put it differently, is there no general relationship between externalities, the kinds of decentralized organizations of society, and the dilemmas, contradictions, or inconsistencies mentioned?

That such a general relationship may exist is also suggested by other work. More than fifteen years ago, Buchanan [1962] and Buchanan and Tullock [1965, pp. 60–62] stressed the fact that collective decision-making not requiring unanimity may lead to negative externalities. If, let us say, a simple majority outvotes a minority on a substantive issue, then the execution of the decision taken will lead to a worse position for the members of the minority. Here we may speak of externalities of collective decision-making, which are again clearly related to the organization of society and to the kind of decision rule used, even if these externalities appear to be different from those considered in the market-failure literature.

It is the purpose of the present paper to explore more closely the relationship between externalities and the possible shortcomings,

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dilemmas, and inconsistencies of decentralized organizational systems.

In Section II we shall introduce a notation that can be used for very different kinds of decentralized societies. Besides, a rather broad definition of externalities will be proposed covering both the traditional and the Buchanan-Tullock types. In Section III, we shall give a proof that externalities are a necessary condition for the existence of cyclical social preferences. This result may in a sense not appear to be too impressive, taking into account the broad definition of externalities to be used. But the relationship has never been stated before. In Section IV we shall discuss the meaning of the result and of the definition of externalities proposed.

II. Definitions and Assumptions

Let us consider a set U of $n > 2$ outcomes and a set V of $m \geq 2$ individuals forming a kind of group or a society. Assume that the right to decide among any pair of outcomes $x_i, x_j \in U$ has been allocated to a subset $V_{ij} \subseteq V$ of society, with $i \neq j; i,j = 1,2, \ldots, n$ and $V_{ij} \neq \phi$.

Assume further that all individuals of society have complete, weak, and transitive preferences. All V_{ij} decide among x_i and x_j following given nonstochastic decision rules such that there exists at least one winning coalition $C_{ij} \subseteq V$, $C_{ij} \neq \phi$, which can decide for society in favor of x_i as against x_j if all of its members prefer the former to the latter, $x_i P_h x_j$ for all $h \in C_{ij}$, whatever be the preferences of all individuals belonging to $V - V_{ij}$ and given those of the people belonging to $V_{ij} - C_{ij}$. Moreover, if C_{ij} is a winning coalition, then $C_{ij} \cup \{g\} \subseteq V_{ij}$ is a winning coalition too, if $x_i P_g x_j$ and if the preferences of all other members of V_{ij} are unchanged.

It is important to realize that the above assumptions allow all kinds of non-oligarchic decentralized institutional arrangements of society, including "pure liberalism" at one extreme, where different individuals have the right to decide among different pairs of all outcomes, and "total direct democracy" at another extreme, where all members of society together have the right to make all the decisions by using simple majority voting.

Next, let us define "social preference relations." We shall say that "society prefers x_i to x_j," $x_i P x_j$, if there exists at least one winning coalition C_{ij} for all of whose members $h \in C_{ij}$ $x_i P_h x_j$ holds. Further, $x_i R x_j$, "society prefers x_i to x_j or is indifferent between them," if there is no winning coalition whose members prefer x_j to x_i; i.e., if $x_j P x_i$ does

not hold. If neither x_iPx_j nor x_jPx_i is valid; i.e., if neither a minimal winning coalition for x_i against x_j nor for x_j against x_i exists, "society is indifferent" between x_i and x_j, x_iIx_j.

We now turn to define externalities. Assume that some winning coalition C_{ij} exists for whose members

$$x_iP_hx_j, \qquad \text{so that } x_iPx_j$$

is valid. Thus, C_{ij} can bring about x_i, whenever x_j is present. But this would lead to externalities for people not belonging to the coalition if either

$$x_iP_hx_j$$

or (and)

$$x_jP_hx_i \qquad \text{for some } h \in V - C_{ij}.$$

In the former case positive and in the latter case negative externalities would be present for some $h \in V - C_{ij}$ if C_{ij} brought about x_i starting from x_j. Thus, the following definition will be used: externalities are absent if for any outcomes $x_i, x_j \in U$, and some C_{ij}, with $x_iP_hx_j$ for all $h \in C_{ij}$, $x_iI_hx_j$ holds for all $h \in V - C_{ij}$. We shall discuss this rather broad definition in Section IV.

III. Proof of the Theorem

We are now able to state and prove Theorem 1.

THEOREM I. Externalities are a necessary condition for the existence of cyclical social preferences.

The proof of the theorem is simple. We assume the existence of cyclical group preferences and the absence of externalities and show that this leads to a contradiction.

Consider outcomes $x_0, x_1, \ldots, x_s$ and assume that

$$(1) \qquad x_sPx_{s-1}P \ldots Px_1Px_0Px_s, \qquad 3 \leq s \leq n$$

holds, such that cyclical social preferences exist. Then there must be winning coalitions $C_{s,s-1}, C_{s-1,s-2}, \ldots, C_{10}, C_{0s}$ for whom

$$(2) \qquad x_0P_hx_s \qquad \text{for all } h \in C_{0s}$$

or (and)

$$(3) \qquad x_iP_hx_{i-1} \qquad \text{for all } h \in C_{i,i-1} \qquad (i = 1,2,\ldots,s)$$

is valid.

It is important to realize that (1) implies

$$\left(\bigcap_{i=1}^{s} C_{i,i-1}\right) \cap C_{0s} = \phi, \tag{4}$$

for if any individual were in all those sets, that individual's preferences would have to be cyclical, i.e. intransitive, contrary to the hypothesis. Note that (4) implies the absence of any oligarchy.

We resume our proof by assuming that no externalities exist for any member of society not belonging to C_{0s} or to the $C_{i,i-1}$ $(i = 1,2, \ldots, s)$. Then it follows from the definition of the absence of externalities and from (1) that

$$x_0 I_h x_s \qquad \text{for all } h \in V - C_{0s} \tag{5}$$

and

$$x_i I_h x_{i-1} \qquad \text{for all } h \in V - C_{i,i-1}, \qquad (i = 1,2, \ldots, s). \tag{6}$$

From (2) and (5) and from (3) and (6) we derive, respectively,

$$x_0 R_h x_s \qquad \text{for all } h \in V \tag{7}$$

and

$$x_i R_h x_{i-1} \qquad \text{for all } h \in V \text{ and all } i = 1,2, \ldots, s. \tag{8}$$

Now consider some $C_{i,i-1}(i = 1,2, \ldots, s)$ or C_{0s} for all of whose members (3) or (2) is valid. Obviously $C_{0s}, C_{i,i-1} \subseteq V$, so that (7) or (8) is true for all $h \in C_{i,i-1}$ or $h \in C_{0s}$. But then one can derive from (2) and (8) or from (3), (7), and (8), respectively,

$$x_s R_h x_{s-1} R_h \ldots R_h x_1 R_h x_0 P_h x_s \qquad \text{for all } h \in C_{0s}, \tag{9}$$

or

$$x_s R_h x_{s-1} R_h \ldots R_h x_i P_h x_{i-1} R_h \ldots R_h x_0 R_h x_s, \qquad \text{for all } h \in C_{i,i-1} \ (i = 1,2, \ldots, s). \tag{10}$$

This result, however, contradicts the assumption of transitive individual preferences. Thus, cyclical social preferences cannot exist if no externalities are present. Q.E.D.

IV. Discussion of the Results and of the Definition of Externalities

Theorem 1 states a remarkable result, even if it depends on a rather broad definition of externalities. Nevertheless, it is not true

that the presence of externalities is sufficient to lead to cyclical social preferences. First, the presence of non-oligarchic, somewhat decentralized, forms of society is necessary to bring about the theorem. Second, even with such an organization, externalities are not a sufficient condition for the existence of cyclical social preferences. It is easy to construct examples in which externalities do not lead to cycles in decentralized organizations of society [Bernholz, 1980]. Third, it can easily be shown that given some preferences of all members of society, there can be different forms of decentralized organizations, some with cyclical social preferences, others without them.

It should be understood that externalities are not only a necessary condition for Arrow's General Impossibility Theorem, but also for other impossibility or dilemma results. As has been shown, e.g., in a recent paper [Bernholz, 1980], Sen's dilemma of a Paretian Liberal is true for all the non-oligarchic decentralized organizations considered here, if enough outcomes are present, and implies cyclical social preferences.

Thus, it follows that the "contradiction between minimal decentralization (including minimal liberalism) and Pareto optimality" also depends on the presence of externalities.

Let us now turn to the definition of externalities. It is true that the absence of externalities in the sense of the broad definition used requires that all members of society outside the winning coalition be indifferent as to the pair of outcomes in question. This may appear to be a stricter requirement than can ever be satisfied in real life, but in fact it is frequently met. In Sen's example, Mr. A (presumably a bachelor) prefers sleeping on his back to sleeping on his stomach, and it is reasonable to assume that every other person in society is indifferent. Clearly, such instances abound. But one has to realize that if this condition is not fulfilled, then there are people in society who will either be hurt or will benefit by a decision taken by the respective winning coalition according to *their own judgments.* Their positions are changed either because they are not members of the decision-making subset, or because they, as members of this subset, are "outvoted," so that externalities in the sense of Buchanan [1962] and Buchanan and Tullock [1965] are present.

We have stressed that our criterion for the presence of externalities has been the *judgments of the people concerned.* We have done so because the same criterion is used when applying the Pareto principle. Thus, whereas the definition of externalities used may be rather broad, a narrower one would imply the abandonment of the Pareto principle, which is widely used in Welfare Economics and is

necessary to establish the above-mentioned theorems, proved by Arrow and Sen. For this principle, too, requires the judgment of everybody, whether he himself prefers some outcomes to others or not.

An extensive discussion of the merits of different concepts of externalities must wait for another, more philosophical, paper. Here, let me first compare our definition to a more traditional one, namely that a (negative) externality exists, whenever there is an individual h such that $x_j P_h x_i$ and a winning coalition exists for each of whose members k, $x_i P_k x_j$ holds. It is obvious that our definition includes the latter one. The definition used is, however, broader in the sense that it also takes into account positive externalities. Moreover, we have preferred to define not the presence but the absence of externalities.

Next I would like to mention two different possibilities to define externalities and the difficulties related to them. One of them would be a definition including only the cases in which "the surroundings" of an individual including himself are phsyically changed by the decisions of others. There are several difficulties with this definition. What are "the surroundings" of an individual? When are they "physically changed"? And should we include in the definition changes in which the respective member of society does not perceive the physical changes or considers them to be irrelevant to his well-being?

A second possibility would be to include the individual perceptions of a wanted or unwanted physical change. But this would not remove the other two difficulties. Are the physical surroundings changed if the individual in question has to endure the sight of somebody in red trousers, which he hates? Certainly the light-waves perceived have been changed when the other person moved into his sight. But is this to be considered a physical change?

This discussion shows that it is not easy to give up the broad definition used in this paper. The result may thus be helpful to concentrate attention on two questions: (1) what kinds of externalities should be taken into account in which decentralized societies; and (2) which allocations of decision rights to which subsets of society together with which decision rules would be most adequate, given different profiles of preferences of the members of society?

V. SUMMARY

Cyclical social preferences in decentralized non-oligarchic societies are a consequence of the existence of three or more different

winning coalitions with no member of society in common. I have shown that externalities are a necessary condition for the presence of cyclical social preferences and are, therefore, the reason for the validity of the General Impossibility Theorem, the Paradox of Liberalism (more generally, of decentralized systems) and of Pareto inferior solutions. A short discussion of the definition of externalities concludes the paper.

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